Gravitational waves from magnetohydrodynamic turbulence in the early-universe

Action Dark Energy 2020 (Oct. 13–15)

Alberto Roper Pol
Postdoctoral Researcher

Laboratoire Astroparticule et Cosmologie (APC)

October 14, 2020

A. Roper Pol *et al.*, *Geophys. Astrophys. Fluid Dyn.* **114**, 130. arXiv:1807.05479 (2020)

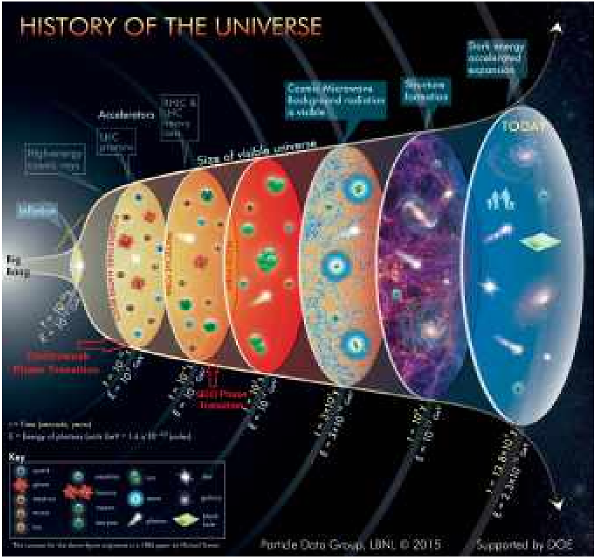
A. Roper Pol *et al.*, *Phys. Rev. D* **102**, 083512. arXiv:1903.08585 (2020)

A. Neronov, A. Roper Pol, C. Caprini, D. Semikoz. arXiv:2009.14174 (2020)

Introduction and Motivation

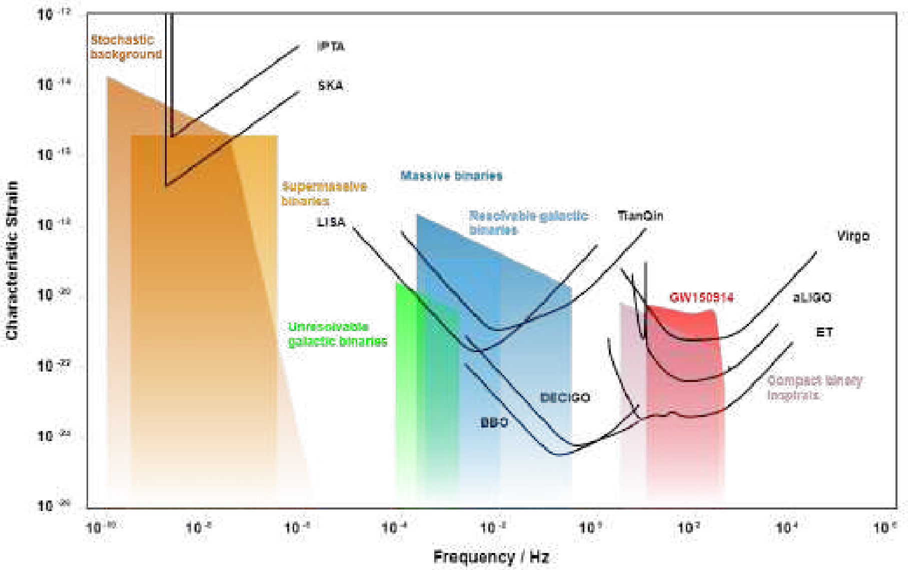
- Generation of cosmological gravitational waves (GWs) during phase transitions and inflation
  - **Electroweak phase transition** ~ 100 GeV
  - **Quantum chromodynamic (QCD) phase transition** ~ 100 MeV
  - Inflation

Introduction and Motivation



Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during phase transitions and inflation
  - Electroweak phase transition ~ 100 GeV
  - Quantum chromodynamic (QCD) phase transition ~ 100 MeV
  - Inflation
- GW radiation as a probe of early universe physics
- Possibility of GWs detection with
  - **Space-based GW detector LISA**
  - **Pulsar Timing Arrays (PTA)**
  - B-mode of CMB polarization


Characteristic Strain
Frequency / Hz
Stochastic background
IPTA
SKA
Supermassive binaries
Massive binaries
LISA
Resolvable galactic binaries
TianQin
Virgo
GW150914
aLIGO
ET
Unresolvable galactic binaries
DECIGO
BBO
Compact binary inspirals

Introduction and Motivation

LISA

- Laser Interferometer Space Antenna (LISA) is a space–based GW detector
- LISA is planned for 2034
- LISA was approved in 2017 as one of the main research missions of ESA
- LISA is composed by three spacecrafts in a distance of 2.5M km

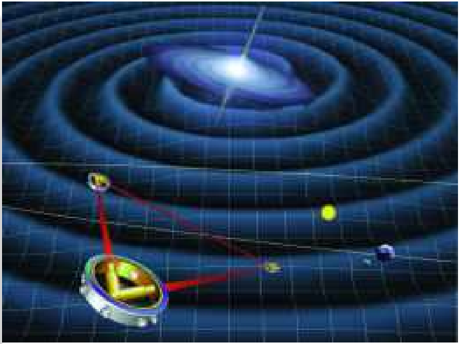
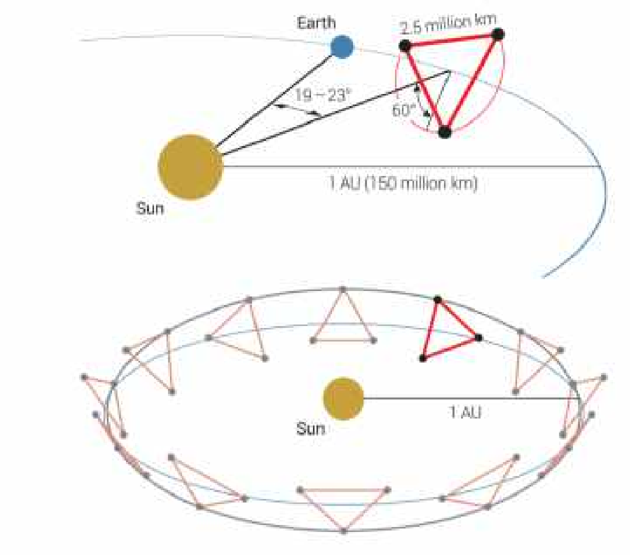


Figure: Artist's impression of LISA from Wikipedia

Orbit of LISA



Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during phase transitions and inflation
  - Electroweak phase transition ~ 100 GeV
  - Quantum chromodynamic (QCD) phase transition ~ 100 MeV
  - Inflation
- GW radiation as a probe of early universe physics
- Possibility of GWs detection with
  - Space-based GW detector LISA
  - Pulsar Timing Arrays (PTA)
  - B-mode of CMB polarization
- Magnetohydrodynamic (MHD) sources of GWs:
  - Hydrodynamic turbulence from phase transition bubbles nucleation
  - Primordial magnetic fields
- Numerical simulations using PENCIL CODE to solve:
  - Relativistic MHD equations
  - Gravitational waves equation

MHD description

Right after the electroweak phase transition we can model the plasma using continuum MHD

- Quark-gluon plasma (above QCD scale)
- Charge-neutral, electrically conducting fluid
- Relativistic magnetohydrodynamic (MHD) equations
- Ultrarelativistic equation of state

$$p = \rho c^2/3$$

- Friedmann–Lemaître–Robertson–Walker model

$$g_{\mu\nu} = \mathrm{diag}\{-1, a^2, a^2, a^2\}$$

Stress–energy tensor

Contributions to the stress-energy tensor

$$T^{\mu\nu} = \left(p/c^2 + \rho\right) U^\mu U^\nu + p g^{\mu\nu} + F^{\mu\gamma} F^{\nu}{}_{\gamma} - \frac{1}{4} g^{\mu\nu} F_{\lambda\gamma} F^{\lambda\gamma},$$

- From fluid motions
  $T_{ij} = \left(p/c^2 + \rho\right) \gamma^2 u_i u_j + p\delta_{ij}$
  Relativistic equation of state:
  $p = \rho c^2/3$
- From magnetic fields:
  $T_{ij} = -B_i B_j + \delta_{ij} B^2/2$

- 4–velocity $U^\mu = \gamma(c, u^i)$
- 4–potential $A^\mu = (\phi/c, A^i)$
- 4–current $J^\mu = (c\rho_{\rm e}, J^i)$
- Faraday tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

MHD equations

Conservation laws

$$T^{\mu\nu}{}_{;\nu} = 0$$

Relativistic MHD equations are reduced to[1]

MHD equations

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}\left(\nabla \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \ln \rho\right) + \frac{1}{\rho c^2}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right]$$

$$\frac{D\boldsymbol{u}}{Dt} = \frac{1}{3}\mathbf{u}\left(\nabla \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \ln \rho\right) - \frac{\boldsymbol{u}}{\rho c^2}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta J^2\right] - \frac{1}{4}c^2 \nabla \ln \rho + \frac{3}{4\rho}\boldsymbol{J} \times \boldsymbol{B} + \frac{2}{\rho}\nabla \cdot (\rho \nu \boldsymbol{S})$$

for a flat expanding universe with comoving and normalized
$p = a^4 p_{\rm phys}, \rho = a^4 \rho_{\rm phys}, B_i = a^2 B_{i,_{\rm phys}}, u_i$, and conformal time t.

[1] A. Brandenburg, K. Enqvist, and P. Olesen, *Phys. Rev. D* **54**, 1291 (1996)

MHD equations

Electromagnetic fields are obtained from Faraday tensor as

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}$$

Generalized Ohm's law

$$\mathbf{E} = \eta \mathbf{J} - \mathbf{u} \times \mathbf{B}$$

Maxwell equations

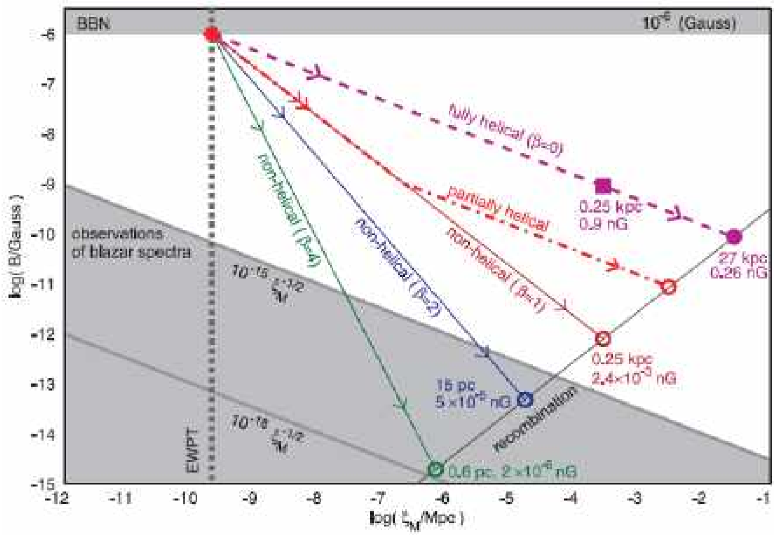
$$\nabla \cdot \mathbf{E} = \rho_{\rm e} c^2, \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mathbf{J} + \cancel{\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}}, \qquad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Maxwell equations + Ohm's law combined:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \boldsymbol{J})$$

Evolution of magnetic strength and correlation length[2]



[2] A. Brandenburg, T. Kahniashvili, S. Mandal, A. Roper Pol, A. Tevzadze, and T. Vachaspati, *Phys. Rev. D* **96**, 123528 (2017)

GWs equation for an expanding flat Universe

- Assumptions: isotropic and homogeneous Universe
- Friedmann–Lemaître–Robertson–Walker (FLRW) metric $\gamma_{ij} = a^2\delta_{ij}$
- Tensor-mode perturbations above the FLRW model:

$$g_{ij} = a^2 \left(\delta_{ij} + h_{ij}^{\rm phys}\right)$$

- GWs equation is[3]

$$\left(\partial_t^2 - \cancel{\frac{a''}{a}} - c^2\nabla^2\right) h_{ij} = \frac{16\pi G}{\mathbf{a} c^2} T_{ij}^{\rm TT}$$

  - h_{ij} are rescaled $h_{ij} = a h_{ij}^{\rm phys}$
  - Comoving spatial coordinates $\nabla = a\nabla^{\rm phys}$
  - Conformal time $\mathrm{d}t = a\ \mathrm{d}t^{\rm phys}$
  - Comoving stress-energy tensor components $T_{ij} = a^4 T_{ij}^{\rm phys}$
  - Radiation-dominated epoch such that $a'' = 0$

[3] L. P. Grishchuk, *Sov. Phys. JETP*, 40, 409-415 (1974)

Normalized GW equation[4]

$$\left(\partial_t^2 - \nabla^2\right) h_{ij} = 6\, T_{ij}^{\mathrm{TT}} / t$$

Properties

- All variables are normalized and non-dimensional
- Conformal time is normalized with t_*
- Comoving coordinates are normalized with c/H_*
- Stress-energy tensor is normalized with $\mathcal{E}_{\mathrm{rad}}^* = 3H_*^2 c^2/(8\pi G)$
- Scale factor is $a_* = 1$, such that $a = t$

[4] A. Roper Pol *et al.*, *Geophys. Astrophys. Fluid Dyn.* **114**, 130. arXiv:1807.05479 (2020)

Gravitational waves equation

Properties

- Tensor-mode perturbations are gauge invariant
- h_{ij} has only two degrees of freedom: h^+, $h^\times$
- The metric tensor is traceless and transverse (TT gauge)

Linear polarization modes $+$ and $\times$

Linear polarization basis (defined in Fourier space)

$$e_{ij}^{+} = (\boldsymbol{e}_1 \times \boldsymbol{e}_1 - \boldsymbol{e}_2 \times \boldsymbol{e}_2)_{ij}$$

$$e_{ij}^{\times} = (\boldsymbol{e}_1 \times \boldsymbol{e}_2 + \boldsymbol{e}_2 \times \boldsymbol{e}_1)_{ij}$$

Orthogonality property

$e_{ij}^{A} e_{ij}^{B} = 2\delta_{AB}$, where $A, B = +, \times$

$+$ and $\times$ modes

$$\tilde{h}^{+} = \frac{1}{2} e_{ij}^{+} \tilde{h}_{ij}^{\rm TT}, \qquad \tilde{T}^{+} = \frac{1}{2} e_{ij}^{+} \tilde{T}_{ij}^{\rm TT}$$

$$\tilde{h}^{\times} = \frac{1}{2} e_{ij}^{\times} \tilde{h}_{ij}^{\rm TT}, \qquad \tilde{T}^{\times} = \frac{1}{2} e_{ij}^{\times} \tilde{T}_{ij}^{\rm TT}$$

Gravitational waves energy density

GWs energy density:

$$\Omega_{\rm GW} = \mathcal{E}_{\rm GW}/\mathcal{E}^0_{\rm crit}, \quad \mathcal{E}^0_{\rm crit} = \frac{3H_0^2c^2}{8\pi G}$$

$$\Omega_{\rm GW} = \int_{-\infty}^{\infty} \Omega_{\rm GW}(k)\,{\rm d}\ln k$$

$$\boldsymbol{\Omega}_{\rm GW}(\mathbf{k}) = (a_*/a_0)^4 \frac{k}{6H_0^2} \int_{4\pi} \left(\left| \dot{\tilde{h}}_+^{\rm phys} \right|^2 + \left| \dot{\tilde{h}}_\times^{\rm phys} \right|^2 \right) k^2 \, {\rm d}\Omega_k$$

$$H_0 = 100\, h_0 \ {\rm km \ s^{-1} \ Mpc^{-1}}$$

$$\frac{a_0}{a_*} \approx 1.254 \cdot 10^{15} \,(T_*/100 \ {\rm GeV})\,(g_{\rm S}/100)^{1/3}$$

Characteristic amplitude of gravitational waves

GWs amplitude:

$$h_{\rm c}^2 = \int_{-\infty}^{\infty} h_{\rm c}^2(k) \, {\rm d} \ln k$$

$$\mathbf{h_{\rm c}^2(k)} = (a_*/a_0) k \int_{4\pi} \left(\left| \tilde{h}_+^{\rm phys} \right|^2 + \left| \tilde{h}_\times^{\rm phys} \right|^2 \right) k^2 \, {\rm d}\Omega_k$$

Frequency:

$$f = H_* (a_*/a_0)(k/2\pi) \approx 1.6475 \cdot 10^{-5} (k/2\pi) \ {\rm Hz}$$

for $T_* = 100$ GeV, $g_{\rm S} \approx g_* = 100$.

Numerical results for decaying MHD turbulence[5]

Initial conditions

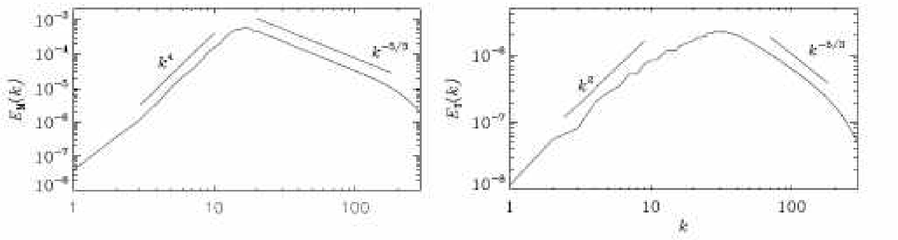
- Fully helical stochastic magnetic field
- Batchelor spectrum, i.e., $E_{\rm M} \propto k^4$ for small k
- Kolmogorov spectrum for inertial range, i.e., $E_{\rm M} \propto k^{-5/3}$
- Total energy density at t_* is $\sim 10\%$ to the radiation energy density
- Spectral peak at $k_{\rm M} = 100 \cdot 2\pi$, normalized with $k_H = H/c$

Numerical parameters

- 1152^3 mesh gridpoints
- 1152 processors
- Wall-clock time of runs is $\sim 1 - 5$ days

[5] A. Brandenburg, *et al. Phys. Rev. D* **96**, 123528 (2017), A. Roper Pol, *et al. Phys. Rev. D* **102**, 083512 (2020)

$k_{\mathrm{M}} = 15$



$$E_{\mathrm{M}}(k) = \frac{1}{2}\int_{4\pi}\left(\tilde{\mathbf{B}}(k)\cdot\tilde{\mathbf{B}}^*(k)\right)k^2 d\Omega$$

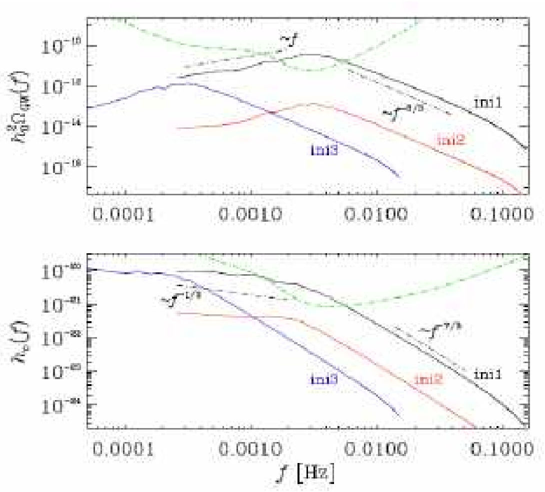
$$\Omega_{\mathrm{M}} = \int_0^\infty E_{\mathrm{M}}(k)dk$$

$$E_{\mathrm{T}}(k) = \int_{4\pi}\left(\tilde{\mathbf{T}}_{ij}(k)\tilde{\mathbf{T}}_{ij}^*(k)\right)k^2 d\Omega$$

$$E_{\mathrm{T}}(k) = \frac{1}{2}\int_{4\pi}\left(\tilde{\mathbf{B}}^2(k)\tilde{\mathbf{B}}^{2,*}(k)\right)k^2 d\Omega$$

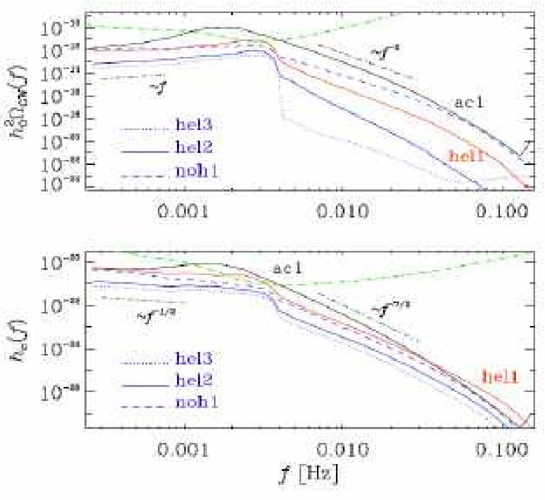
$$\Omega_{\mathrm{T}} = \int_0^\infty E_{\mathrm{T}}(k)dk$$

Numerical results for decaying MHD turbulence



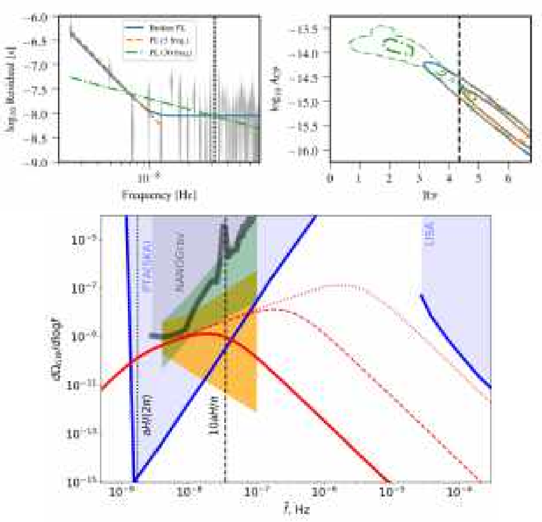
Run	$\mathcal{E}_0, \mathcal{F}_0$	η	$\Omega_s^{\max}$	$\Omega_{\rm GW}^{\rm sat}$	i	hel	$t_{\max}$	N
ini1	—	5e-6	1.16e-01	2.05e-09	M	y	1.00	100
ini2	—	5e-8	7.62e-03	6.38e-12	M	y	1.00	100
ini3	—	5e-7	7.62e-03	6.36e-10	M	y	1.00	10

Forced turbulence (built-up primordial magnetic fields and hydrodynamic turbulence)



Run	$\mathcal{E}_0, \mathcal{F}_0$	η	Ω_T^{max}	Ω_{GW}^{sat}	i	hel	t_{max}	N
hel1	1.4e-3	5e-7	2.17e-02	4.43e-09	M	y	1.10	100
hel2	8.0e-4	5e-7	7.18e-03	4.67e-10	M	y	1.10	100
hel3	2.0e-3	5e-7	4.62e-03	2.09e-10	M	y	1.01	100
hel4	1.0e-2	2e-6	3.49e-03	1.10e-11	M	y	1.01	1000
noh1	1.4e-3	5e-7	1.44e-02	3.10e-09	M	n	1.10	100
noh2	8.0e-4	2e-6	4.86e-03	3.46e-10	M	n	1.10	100
ac1	3.0	2e-5	1.33e-02	5.66e-08	K	n	1.10	100
ac2	3.0	5e-5	1.00e-02	3.52e-08	K	n	1.10	100
ac3	1.0	5e-6	2.87e-03	2.75e-09	K	n	1.10	100

NANOGrav observation QCD phase transition[6]



[6] NANOGrav collaboration, arXiv:2009.04496 (2020)
A. Neronov, A. Roper Pol, C. Caprini, D. Semikoz. arXiv:2009.14174 (2020)

Conclusions

- For some of our simulations we obtain a detectable signal from EWPT by future GW detector LISA.
- GW equation is normalized such that it can be easily scaled for different times within the radiation-dominated epoch
- Novel f spectrum obtained for GWs in high frequencies range vs f^3 obtained from analytical estimates (above horizon scales)
- Bubble nucleation and magnetogenesis physics can be coupled to our equations for more realistic production analysis.
- Potential detection by NANOGrav
- Information on large-scale relic magnetic fields with cosmological origin

The End
Thank You!

roperpol@apc.in2p3.fr